12 CLASS

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CBSE 39 SAMPLE PAPERS

MATHEMATICS (041)

SESSION 2026-27

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SAMPLE PAPER

issued by CBSE for Board Exams (2026-27)

Mathematics (041) - Class 12

Time Allowed : 180 Minutes **Max. Marks : 80**

General Instructions :

1. This Question paper contains **38 questions**. All the questions are compulsory.
2. This Question paper is divided into **five sections - A, B, C, D and E**.
3. Section A has **18 MCQs** and **02 Assertion-Reason (A-R)** based questions of **1 mark** each.
   Section B has **05 Very Short Answer (VSA)-type questions** of **2 marks** each.
   Section C has **06 Short Answer (SA)-type questions** of **3 marks** each.
   Section D has **04 Long Answer (LA)-type questions** of **5 marks** each.
   Section E has **03 Case-study / Source-based / Passage-based** questions with **sub-parts** (with a total of **4 marks** for each Case-study / Source-based / Passage-based question).
4. There is no overall choice. However, **internal choice** has been provided in
   - **02 Questions of Section B**
   - **03 Questions of Section C**
   - **02 Questions of Section D**
   - **02 Questions of Section E**

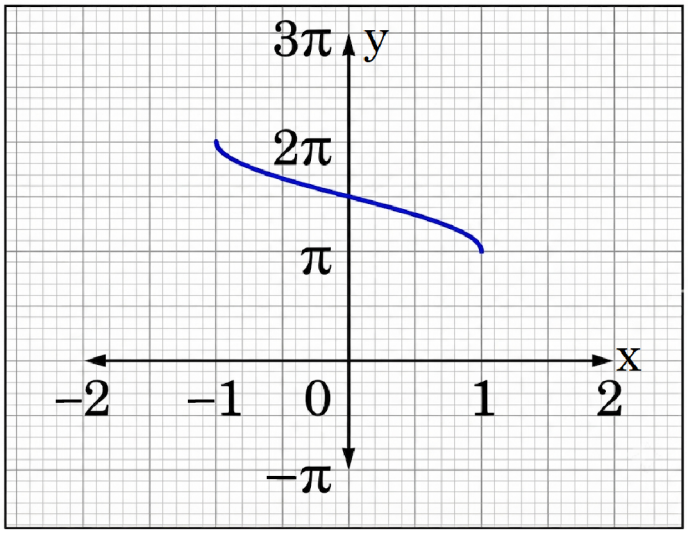
   You have to attempt only one of the alternatives in all such questions.
5. Draw neat and clean figures wherever required. Use of calculator is not allowed.

SECTION A

*(Question numbers 01 to 20 carry **1 mark** each.)*

*Followings are **multiple choice questions**. Select the correct option in each one of them.*

01. Identify the function whose graph is given below.



(a) $\cos^{-1}\left(\frac{x}{2}\right)$ (b) $\cos^{-1}(2x)$ (c) $\pi-\cos^{-1}x$ (d) $\pi+\cos^{-1}x$

01. **FOR VISUALLY IMPAIRED**

Find the range of $\frac{\pi}{2}+\sec^{-1}\left(\frac{x}{2}\right)$.

(a) $[0, \pi]$ (b) $[0, \pi]-\left\{\frac{\pi}{2}\right\}$ (c) $\left[\frac{\pi}{2}, \frac{3\pi}{2}\right]$ (d) $\left[\frac{\pi}{2}, \frac{3\pi}{2}\right]-\{\pi\}$

02. Domain of the function $\sec^{-1}(2x)$ is

(a) $(-\infty, \infty)$ (b) $\left(-\frac{1}{2}, \frac{1}{2}\right)$

(c) $\left(-\infty, -\frac{1}{2}\right)\cup\left(\frac{1}{2}, \infty\right)$ (d) $\left(-\infty, -\frac{1}{2}\right]\cup\left[\frac{1}{2}, \infty\right)$

03. Value of $\cot^{-1}\left(2\cos\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)\right)\right)$ is

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$

04. If $A=\begin{bmatrix} 2 & 1 & -2 \\ 3a & 0 & 4 \\ b & 2c & 1 \end{bmatrix}$ is a symmetric matrix, then value of $\sqrt{6a+2b+3c}$ is

(a) -2 (b) 2 (c) ± 2 (d) 4

05. If P and Q are matrices of the same order such that $P=\begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix}$, $|Q|=3$, then find $|\text{adj}(PQ)|$.

(a) -3 (b) 1 (c) 6 (d) 36

06. If $\begin{bmatrix} 1 & x \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} 4 \\ -1 \end{bmatrix}=O$, then value of x is

(a) -2 (b) -1 (c) 1 (d) 2

07. The derivative of $\cos^{-1}(2x^2-1)$ with respect to $\cos^{-1}(x)$; $0 \le x \le 1$ is

(a) 2 (b) ± 1 (c) $\frac{1}{2}$ (d) -2

08. If a, b, c are arbitrary constants such that the function defined by $f(x)=\begin{cases} ax^2+b, & b\neq 0,\ x\le 1 \\ bx^2+ax+c, & x>1 \end{cases}$ is differentiable at $x=1$, then which of the following could be a triplet for (a, b, c)?

(a) (3, 2, 0) (b) (0, 0, 3) (c) $\left(\frac{3}{2}, 2, 0\right)$ (d) $\left(3, \frac{3}{2}, 0\right)$

09. If $y=a\left\{x+2a\sin\left(\frac{x}{a}\right)\right\}$, $a>0$, $x\in[0, \pi a]$; is a decreasing function, then the **largest** interval in which x lies is

(a) $\left(\frac{5\pi}{6}, \pi\right]$ (b) $\left(\frac{2\pi}{3}, \pi\right]$ (c) $\left(\frac{5\pi a}{6}, \pi a\right]$ (d) $\left(\frac{2\pi a}{3}, \pi a\right]$

10. The values of x for which the derivative of the function $y=x^4$ is greater than the value of the function are

(a) $0<x<4$ (b) $0<x\le 4$ (c) $x<0$ or $x>4$ (d) $x<0$ or $x\ge 4$

11. The degree and order of the differential equation $y-x\left(\frac{dy}{dx}\right)^3=a\left(y+\frac{d^2y}{dx^2}\right)^2$ are respectively

(a) 3, 1 (b) 2, 2 (c) 3, 2 (d) 4, 2

12. If $\int \frac{1}{1+e^{2x}}dx = x - P\log(1+e^{2x}) + C$, then 'P' is equal to

(a) $-\frac{1}{2}$ (b) $-\frac{1}{3}$ (c) $\frac{1}{3}$ (d) $\frac{1}{2}$

13. If vector $2\hat{i} - u\hat{j} + 3\hat{k}$ has magnitude 4, then the value of 'u' is

(a) 1 (b) $\sqrt{2}$ (c) $\pm\sqrt{3}$ (d) ± 2

14. Angle made by the vector $\hat{i} + \hat{j} + \sqrt{2}\,\hat{k}$ with the positive direction of the z-axis is

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$

15. A vector perpendicular to the vectors $\hat{i} + \hat{j}$ and $\hat{i} - \hat{j}$ is

(a) $-2\hat{j}$ (b) $3\hat{k}$ (c) $2\hat{i}$ (d) $2\hat{i} + \hat{k}$

16. The probability that it rains on a particular day is $\frac{1}{5}$. Assuming weather conditions of any two days are independent, find the probability that it rains on three consecutive days.

(a) $\frac{4}{5}$ (b) $\frac{1}{5}$ (c) $\frac{1}{25}$ (d) $\frac{1}{125}$

17. A die is thrown twice. What is the probability that at least one number is 4, given the sum of the numbers appearing is a multiple of 5?

(a) $\frac{3}{5}$ (b) $\frac{4}{5}$ (c) $\frac{3}{7}$ (d) $\frac{4}{7}$

18. A and B are two independent events such that $P(A) = \frac{1}{6}$, $P(B) = k$. If the probability of occurrence of exactly one of them is $\frac{1}{3}$, then find the value of k.

(a) $\frac{1}{4}$ (b) $\frac{2}{3}$ (c) $\frac{3}{4}$ (d) $\frac{5}{6}$

*Followings are **Assertion-Reason based questions**.*

In the following questions (Question numbers 19 and 20), a statement of Assertion (A) is followed by a statement of Reason (R).

Select the correct answer out of the following choices.

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).
(b) Both Assertion (A) and Reason (R) are true and Reason (R) is not the correct explanation of Assertion (A).
(c) Assertion (A) is true but Reason (R) is false.
(d) Assertion (A) is false but Reason (R) is true.

19. **Assertion (A) :** The value of the integral $\int_{-1}^{1} |x-2|\,dx$ is 4.

Reason (R) : If $f(-x) = f(x)$, then $\int_{-a}^{a} f(x)\,dx = 2\int_{0}^{a} f(x)\,dx$.

20. **Assertion (A) :** The general solution of a differential equation represents a family of curves.
Reason (R) : The derivative of an arbitrary constant is zero.

SECTION B

*(Question numbers 21 to 25 carry **2 marks** each.)*

21. (A) Evaluate $\int \frac{1}{x\sqrt{x^6 - 1}}dx$.

OR

(B) Evaluate $\int \frac{\cos x - \sin x}{\sqrt{\sin 2x}} dx$.

22. If $y = \sin(x + y)$ and $\frac{dy}{dx} = 0$ (at a point), then prove that $y = \pm 1$.

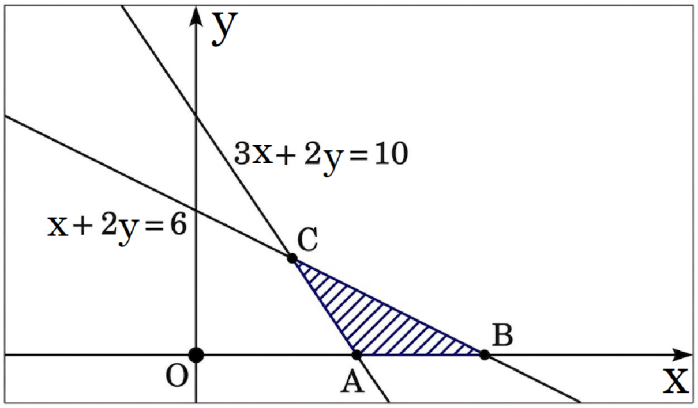
23. (A) A line passing through the points (2, 6, 5), and (3, p, 7) is parallel to the line joining the points (–4, 3, 4) and (–1, 0, q). Find the value of $\frac{q}{p}$.

OR

(B) A rectangle ABCD is such that $\overrightarrow{AC} = \vec{u}$, and $\overrightarrow{BD} = \vec{v}$, then find the vectors representing the four sides of the rectangle (in cyclic order starting from $\overrightarrow{AB}$).

24. For two non-zero vectors $\vec{a}$ and $\vec{b}$, $|\vec{a}| = \sqrt{6}$, $\vec{a}.\vec{b} = 1$ and $\vec{b} \times \vec{a} = 3\hat{i} + 4\hat{j} - 2\hat{k}$. Find $|\vec{b}|$.

25. The feasible region of a linear programming problem is shown in the given figure.



Write all the constraints on the decision variables involved.

25. **FOR VISUALLY IMPAIRED**

For a linear programming problem, the maximum value of an objective function $Z = 3x + 4y$ exists at the points (p, q) and $\left(2p, \frac{q}{2}\right)$. Find the relation between p and q.

SECTION C

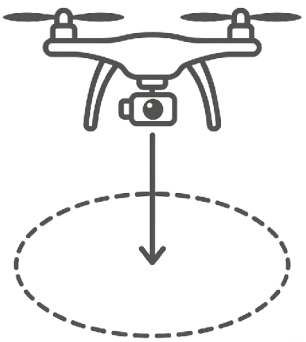
*(Question numbers 26 to 31 carry **3 marks** each.)*

26. For two matrices $P = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 2 \end{bmatrix}$ and $Q = \begin{bmatrix} 1 & 0 \\ 0 & 2 \\ 1 & 0 \end{bmatrix}$, find PQ and QP, whichever is defined. Also, find the inverse of the product matrix, if it exists.

27. The circular area of vision on the ground by a surveillance drone fitted with camera increases at the rate of 44 m^2 per metre as it gains vertical height.

The radius of the circle on the ground increases at the rate of 6 m/s. Find the rate at which the height of the drone increases with time, at the instant when the radius of the circular area on the ground is 14 m.

$\left(\text{Use } \pi = \frac{22}{7}\right)$.



28. (A) If $x = \sin^3 t$, $y = \cos^3 t$, evaluate $\frac{d^2y}{dx^2}$ at $\left(\frac{1}{2\sqrt{2}}, \frac{1}{2\sqrt{2}}\right)$.

OR

(B) If $y = x^{\log x} + (\log x)^x$; $x > 1$, then evaluate $\frac{dy}{dx}$.

29. (A) Find the particular solution of the differential equation $x\,dy + (y - x^2e^x)\,dx = 0$, given that $y(1) = 0$.

OR

(B) Find the particular solution of the differential equation $(x^2 + y^2)\,dy = xy\,dx$, given that $y(0) = 1$.

30. (A) Find the foot of the perpendicular drawn from the point (2, 1, 4) on the following line: $x - 2 = \frac{2y+3}{4} = 2 - z$. Also find the equation of the line passing through this point and the point (2, 1, 4).

OR

(B) Find the angle between the lines: $\frac{x-1}{2} = \frac{y-1}{2} = \frac{z-2}{1}$ and $\frac{x-5}{2} = y - \frac{1}{2} = \frac{3-z}{2}$. Also find the Cartesian equation of a line passing through the point (1, 1, 1) and perpendicular to both lines.

31. Solve the linear programming problem graphically.

Maximize $Z = 40x + 30y$

Subject to constraints $x + y \le 6$, $2x \ge y$, $y \ge 2$.

31. **FOR VISUALLY IMPAIRED**

For a linear programming problem, vertices of the feasible region are (20, 0), (40, 0), (30, 10), (0, 20) and (0, 10). Minimize and maximize the objective function $Z = 50x - 20y$.

SECTION D

*(Question numbers 32 to 35 carry **5 marks** each.)*

32. (A) Let $A = N \times N$, where N is the set of natural numbers and a relation R is defined on A such that (a, b) R (c, d) if and only if $a^2 + d^2 = b^2 + c^2$. Check whether R is an equivalence relation or not?

OR

(B) A relation $R : (-\infty, \infty) \to [0, \infty)$ is defined as $R = \{(x, y) : y = x^2 + 5\}$. Draw its graph and check whether it is a function or not. If the given relation is a function, then examine whether the function is injective, and whether it is surjective or not.

FOR VISUALLY IMPAIRED

A relation $R : (-\infty, \infty) \to [0, \infty)$ is defined as $R = \{(x, y) : y = x^2 + 7\}$. Check whether it is a function or not. If the given relation is a function, then examine whether the function is injective, surjective, or both, and conclude whether it is bijective or not.

33. Evaluate $\int_{-a}^{a} \sqrt{\frac{a-x}{a+x}}\,dx$.

34. (A) Using integration, find the area of the region $\left\{(x, y) : 0 \le y \le \sqrt{4 - x^2},\ x \ge 2 - y\right\}$.

OR

(B) Using integration, find the area of the region bounded by the curve $y^2 = 2x$ and the line $x - 2y = 0$.

FOR VISUALLY IMPAIRED

34. (A) Using integration, find the area of the region enclosed within $25x^2 + 16y^2 = 400$.

OR

(B) Using integration, find the area of the region in the first quadrant bounded by the curve $y^2 = 8x$, and the lines $x = 2$, $x = 5$ and x-axis.

35. A fitness app syncs with four types of smartwatches: Alpha, Beta, Gamma and Delta. 45% of the users use Alpha, 30% use Beta, 15% use Gamma and 10% use Delta.
Due to device errors, the probabilities of getting a false heart-rate alert from the Alpha, Beta, Gamma and Delta smartwatches are respectively 0.01, 0.02, 0.05, and 0.01.

Based on the given information, answer the following questions.

(i) Find the probability that a randomly selected user gets a false alert.
(ii) If a false alert occurs, what is the probability that it came from a Beta smartwatch?

SECTION E

*(Question numbers 36 to 38 carry **4 marks** each.)*
*This section contains **three Case-study / Passage based questions**.*
*First two questions have **three sub-parts** (i), (ii) and (iii) of **marks 1, 1 and 2** respectively.*
*Third question has **two sub-parts** (i) and (ii) of **2 marks** each.*

36. CASE STUDY I :

Two security sensors in a multi-storey mall emit laser beams for movement detection. Three of these beams travel along the following lines.

$l_1 : \vec{r} = (\lambda + 1)\hat{i} + (3\lambda + 2)\hat{j} + (3 - \lambda)\hat{k}$,

$l_2 : \vec{r} = 2(\mu + 1)\hat{i} + (3\mu - 1)\hat{j} + (\mu + 1)\hat{k}$, and

$l_3 : \vec{r} = (\gamma + 3)\hat{i} + (3\gamma - 1)\hat{j} + (1 - \gamma)\hat{k}$.



Based on the given information, answer the following questions.

(i) Show that the lines l_1 and l_2 are not parallel to each other.

(ii) Show that the lines l_1 and l_3 are parallel to each other.

(iii) (A) Find the shortest distance between the lines l_1 and l_2.

OR

(iii) (B) Find the shortest distance between the lines l_1 and l_3.

37. CASE STUDY II :

Water is essential for life, and rivers are vital water sources across the world. The **Yamuna Action Plan** (YAP), one of India's largest river restoration projects, was launched to clean the Yamuna, but its effectiveness (WQI), the **Water Quality Index** must be measured through cost-benefit, economic and environmental analysis. Therefore, comprehensive pollutant treatment, prevention of further contamination, and community awareness are crucial for sustainable river management.

Implementing the same concept to the river Yamuna, if an initial base line WQI is 35, a quadratic expression is formed to measure the impact of investment of ₹x crore on the WQI i.e., W(x) as $W(x) = 35 + 1.6x - 0.02x^2$, where x is in Rupees in crores.



Use the given W(x) to answer the following questions.

(i) Find the value of x at the stationary point for the function W(x).

(ii) Find the maximum value of the WQI using the second derivative test.

(iii) (A) If the investment is ₹35 crores, then find by how much the WQI is short of its maximum achievable value.

OR

(iii) (B) Evaluate the WQI when the investment is ₹60 crores. Determine whether this indicates an enhancement or a diminishing return in our objective when compared with the optimum WQI.

38. CASE STUDY III :

To celebrate their child's birthday, a family decided to distribute fruits among children in three orphanages. The fruit vendor packed three different types of baskets, according to the quantity required at each orphanage, details are given as follows.

Basket A contains 5 kg Apples, 10 kg Oranges and 5 kg Pomegranate
Basket B contains 10 kg Apples, 10 kg Oranges and 5 kg Pomegranate
Basket C contains 8 kg Apples, 15 kg Oranges and 7 kg Pomegranate

Cost of basket A, B and C are respectively ₹2750, ₹3500, and ₹4100.



Based on the given information, answer the following questions.

(i) Write the matrix form of the equations involved in this situation and find the adjoint of the coefficient matrix involved.

(ii) Using inverse of the matrix, calculate the price of each fruit per kg.

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Note : We have **re-typed** the Official CBSE Sample Question Paper and, also added the **necessary illustrations / corrections** at some places.

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 # O.P. GUPTA

Indira Award Winner & Math Mentor

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🕮 Detailed Solutions for CBSE Sample Paper (*Session 2026-27*)

SECTION A

01. (d) Recall that, for $-1 \le x \le 1$, $0 \le \cos^{-1} x \le \pi$.

So, clearly, $\pi \le (\pi + \cos^{-1} x) \le 2\pi$.

Hence, the function is $\pi + \cos^{-1} x$ whose graph is given.
Note that, the range shown in the graph is $[\pi, 2\pi]$.

01. **FOR VISUALLY IMPAIRED**

(d) Since $0 \le \sec^{-1}\left(\frac{x}{2}\right) \le \pi$, $\sec^{-1}\left(\frac{x}{2}\right) \ne \frac{\pi}{2}$.

So, $\frac{\pi}{2} \le \frac{\pi}{2} + \sec^{-1}\left(\frac{x}{2}\right) \le \frac{3\pi}{2}$, $\frac{\pi}{2} + \sec^{-1}\left(\frac{x}{2}\right) \ne \pi$.

Hence, the range of $\frac{\pi}{2} + \sec^{-1}\left(\frac{x}{2}\right)$ is $\left[\frac{\pi}{2}, \frac{3\pi}{2}\right] - \{\pi\}$.

02. (d) Recall that, $\sec^{-1}(x)$ is defined for all $x \in \mathbb{R} - (-1, 1)$.

Therefore, $\sec^{-1}(2x)$ will be defined for all $2x \in \mathbb{R} - (-1, 1)$ i.e., $x \in \mathbb{R} - \left(-\frac{1}{2}, \frac{1}{2}\right)$.

Hence, the domain of the function $\sec^{-1}(2x)$ is $x \in \left(-\infty, -\frac{1}{2}\right] \cup \left[\frac{1}{2}, \infty\right)$.

03. (a) $\cot^{-1}\left(2\cos\left(\frac{1}{2}\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)\right)\right) = \cot^{-1}\left(2\cos\left(\frac{1}{2}\times\frac{\pi}{3}\right)\right) = \cot^{-1}\left(2\cos\left(\frac{\pi}{6}\right)\right)$

$= \cot^{-1}\left(2\times\frac{\sqrt{3}}{2}\right) = \cot^{-1}\sqrt{3} = \frac{\pi}{6}$.

04. (b) Since A is a symmetric matrix so, we must have $a_{ij} = a_{ji}$.

That is, $a_{21} = a_{12} \Rightarrow 3a = 1$; $a_{31} = a_{13} \Rightarrow b = -2$ and $a_{32} = a_{23} \Rightarrow 2c = 4$.

Hence, $\sqrt{6a + 2b + 3c} = \sqrt{2(3a) + 2(b) + \frac{3}{2}(2c)} = \sqrt{2\times1 + 2(-2) + \frac{3}{2}(4)} = \sqrt{2 - 4 + 6} = \sqrt{4} = 2$.

05. (c) It is given that P and Q are matrices of the same order.
As the order of P is 2, so the order of Q will also be 2.
Clearly, the order of (PQ) will be 2 as well.
Now $|\text{adj}(PQ)| = |(PQ)|^{2-1} = |(PQ)| = |P||Q| = (2\times1 - 0\times1)\times3 = 2\times3 = 6$.

06. (d) $\begin{bmatrix}1 & x\end{bmatrix}\begin{bmatrix}1 & 0\\ 0 & 2\end{bmatrix}\begin{bmatrix}4\\ -1\end{bmatrix} = O$

$\Rightarrow \begin{bmatrix}1 & 2x\end{bmatrix}\begin{bmatrix}4\\ -1\end{bmatrix} = O$

$\Rightarrow [4 - 2x] = [0]$

By equality of matrices, we get $4 - 2x = 0$
$\Rightarrow -2x = -4$
$\therefore x = 2$.

07. (a) Let $u = \cos^{-1}(x) \Rightarrow x = \cos u$. As $0 \le x \le 1$ so, $0 \le \cos u \le 1 \Rightarrow 0 \le u \le \frac{\pi}{2}$.

Now let $v = \cos^{-1}(2x^2 - 1)$

$\Rightarrow v = \cos^{-1}(2\cos^2 u - 1) = \cos^{-1}(\cos 2u)$

Since $0 \le 2u \le \pi$, so $v = 2u$

Hence, $\dfrac{dv}{du} = 2$.

08. (d) As f is differentiable at $x = 1$ so, it will be continuous also at same point $x = 1$.

Now $f(1) = a + b$; $\lim\limits_{x \to 1^+} f(x) = \lim\limits_{x \to 1^+} (bx^2 + ax + c) = b + a + c$.

Taking $f(1) = \lim\limits_{x \to 1^+} f(x)$, we get $a + b = b + a + c$

$\Rightarrow c = 0$.

Also $f'(x) = \begin{cases} 2ax, & x \le 1 \\ 2bx + a, & x > 1 \end{cases} \Rightarrow f'(1) = \begin{cases} 2a, & x \le 1 \\ 2b + a, & x > 1 \end{cases}$

As f is differentiable at $x = 1$ so, $Lf'(1) = Rf'(1)$.

That is, $2a = 2b + a$

$\Rightarrow a = 2b$.

Hence, (a, b, c) becomes (2b, b, 0) which is satisfied by option (d) $\left(3, \dfrac{3}{2}, 0\right)$.

09. (d) Since the function is decreasing so, $\dfrac{dy}{dx} \le 0$

$\Rightarrow a\left\{1 + 2\cos\left(\dfrac{x}{a}\right)\right\} \le 0$

As $a > 0$ so, we must have $1 + 2\cos\left(\dfrac{x}{a}\right) \le 0 \qquad \Rightarrow \cos\left(\dfrac{x}{a}\right) \le -\dfrac{1}{2}$

That is, $-1 \le \cos\left(\dfrac{x}{a}\right) \le -\dfrac{1}{2}$

$\Rightarrow \pi \ge \left(\dfrac{x}{a}\right) \ge \dfrac{2\pi}{3}$

$\Rightarrow \pi a \ge x \ge \dfrac{2\pi a}{3}$

Therefore, $x \in \left[\dfrac{2\pi a}{3}, \pi a\right]$.

Clearly, the **largest** interval from the given options will be $x \in \left(\dfrac{2\pi a}{3}, \pi a\right]$.

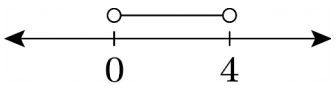
10. (a) As $\dfrac{dy}{dx} > y$

So, $4x^3 > x^4$

$\Rightarrow 4x^3 - x^4 > 0$

$\Rightarrow x^3(4 - x) > 0$

Clearly, $0 < x < 4$.



11. (b) The highest order derivative is $\dfrac{d^2y}{dx^2}$ and it has the exponent of 2 (once we expand the RHS).

Hence, the degree and order of the given differential equation are respectively 2, 2.

12. (d) As $\displaystyle\int \frac{1}{1 + e^{2x}}dx = \int\left(1 - \frac{1}{2} \times \frac{2e^{2x}}{1 + e^{2x}}\right)dx = x - \frac{1}{2}\log\left|1 + e^{2x}\right| + C \quad \left[\because \int \frac{f'(x)\,dx}{f(x)}dx = \log|f(x)| + C\right.$

On comparing $\int \frac{1}{1+e^{2x}}dx = x - \frac{1}{2}\log\left|1+e^{2x}\right| + C$ with $\int \frac{1}{1+e^{2x}}dx = x - P\log(1+e^{2x}) + C$, we shall get $P = \frac{1}{2}$.

13. (c) $\left|2\hat{i} - u\hat{j} + 3\hat{k}\right| = 4$

$\Rightarrow \sqrt{4+u^2+9} = 4$

On squaring both the sides, we get $13 + u^2 = 16$

$\Rightarrow u^2 = 3$

$\therefore u = \pm\sqrt{3}$.

14. (b) For $\hat{i}+\hat{j}+\sqrt{2}\hat{k}$, the unit vector is $\frac{\hat{i}+\hat{j}+\sqrt{2}\hat{k}}{\sqrt{1+1+2}} = \frac{1}{2}\hat{i} + \frac{1}{2}\hat{j} + \frac{\sqrt{2}}{2}\hat{k}$.

The angle made by $\hat{i}+\hat{j}+\sqrt{2}\hat{k}$ with the positive direction of the z-axis is $\cos\gamma = \frac{\sqrt{2}}{2}$.

Therefore, $\cos\gamma = \frac{1}{\sqrt{2}}$

$\Rightarrow \gamma = \frac{\pi}{4}$.

15. (b) Note that, both the given vectors $\hat{i}+\hat{j}$ and $\hat{i}-\hat{j}$ are in xy-plane.

So, a vector perpendicular to both must be in the direction of z-axis. Hence, $3\hat{k}$ is the required vector.

Alternatively, the vector $\perp$ to both $\hat{i}+\hat{j}$ and $\hat{i}-\hat{j} = (\hat{i}+\hat{j})\times(\hat{i}-\hat{j}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 0 \\ 1 & -1 & 0 \end{vmatrix} = (-2\hat{k})$.

Any scalar multiple of $(-2\hat{k})$ is also perpendicular to both the given vectors. Observing all the options, we notice that $3\hat{k}$ is a scalar multiple of $(-2\hat{k})$.

16. (d) Probability that it rains on a particular day is $\frac{1}{5}$.

Therefore, the probability that it rains on three consecutive days $= \frac{1}{5}\times\frac{1}{5}\times\frac{1}{5} = \frac{1}{125}$.

17. (d) Let A : sum of the numbers appeared is a multiple of 5, B : at least one number is 4.
Then $A = \{(1,4),(2,3),(3,2),(4,1),(4,6),(5,5),(6,4)\}$; $A \cap B = \{(1,4),(4,1),(4,6),(6,4)\}$.

$\therefore P(B \mid A) = \frac{P(B\cap A)}{P(A)} = \frac{\frac{4}{36}}{\frac{7}{36}} = \frac{4}{7}$.

18. (a) Probability of occurrence of exactly one of them $= \frac{1}{3}$

$\Rightarrow P(A\cap\bar{B}) + P(\bar{A}\cap B) = \frac{1}{3}$

$\Rightarrow P(A).P(\bar{B}) + P(\bar{A}).P(B) = \frac{1}{3}$

If A and B are two independent events, then A and $\bar{B}$ as well as $\bar{A}$ and B are independent events.

$$\Rightarrow \frac{1}{6}.(1-k)+\frac{5}{6}.(k)=\frac{1}{3}$$
$$\Rightarrow (1-k)+5k=2$$
$$\Rightarrow 4k=1$$
$$\Rightarrow k=\frac{1}{4}.$$

19. (b) $\int_{-1}^{1}|x-2|dx=\int_{-1}^{1}-(x-2)dx=-\left[\frac{(x-2)^2}{2\times 1}\right]_{-1}^{1}=-\frac{1}{2}[1-9]=4$; so Assertion (A) is true.

Also, the statement given in Reason (R) is also true; however it is not the correct explanation of Assertion (A).

20. (a) **Explanation for Assertion (A) :** Note that, on solving the differential equation, an arbitrary constant appears in the general solution. Different values of the arbitrary constant give different curves, which together constitute a **family of curves**. So, Assertion is true.

Explanation for Reason (R) : Because the derivative of each arbitrary constant vanishes, every member of the family (i.e., every choice of the constants) satisfies the same differential equation. Consequently the general solution, which retains those arbitrary constants, represents the entire family of solution curves. So, Reason (R) is also true.

Hence, both the statements given in Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

SECTION B

21. (A) Let $I=\int\frac{1}{x\sqrt{x^6-1}}dx$

$$\Rightarrow I=\frac{1}{6}\int\frac{6x^5}{x^6\sqrt{x^6-1}}dx$$

Put $x^6-1=t^2 \Rightarrow 6x^5dx=2t\,dt$.

$$\therefore I=\frac{1}{6}\int\frac{2t}{(t^2+1)\sqrt{t^2}}dt$$
$$\Rightarrow I=\frac{1}{6}\int\frac{2}{(t^2+1)}dt=\frac{2}{6}\tan^{-1}t+C$$
$$\therefore I=\frac{1}{3}\tan^{-1}\sqrt{x^6-1}+C.$$

OR

(B) Let $I=\int\frac{\cos x-\sin x}{\sqrt{\sin 2x}}dx$

Put $\sin x+\cos x=t \Rightarrow (\cos x-\sin x)dx=dt$.

Also $(\sin x+\cos x)^2=t^2$

$$\Rightarrow \sin^2 x+\cos^2 x+2\sin x\cos x=t^2$$
$$\Rightarrow \sin 2x=t^2-1 \qquad [\because \sin^2 x+\cos^2 x=1$$
$$\therefore I=\int\frac{dt}{\sqrt{t^2-1}}$$
$$\Rightarrow I=\log\left|t+\sqrt{t^2-1}\right|+C$$

$\Rightarrow I = \log\left|(\sin x + \cos x) + \sqrt{(\sin x + \cos x)^2 - 1}\right| + C$

$\therefore I = \log\left|(\sin x + \cos x) + \sqrt{\sin 2x}\right| + C$.

22. $y = \sin(x + y)$

$\Rightarrow \sin^{-1} y = (x + y)$

Differentiating w.r.t. y both the sides, we get $\frac{1}{\sqrt{1-y^2}} = \frac{dx}{dy} + 1$

$\Rightarrow \frac{1}{\sqrt{1-y^2}} - 1 = \frac{dx}{dy}$

$\Rightarrow \frac{dy}{dx} = \frac{\sqrt{1-y^2}}{1-\sqrt{1-y^2}}$

As $\frac{dy}{dx} = 0$, so $\frac{\sqrt{1-y^2}}{1-\sqrt{1-y^2}} = 0$

$\Rightarrow \sqrt{1-y^2} = 0$

$\Rightarrow 1 - y^2 = 0$

$\Rightarrow y^2 = 1$

$\therefore y = \pm 1$.

Alternatively, we have $y = \sin(x + y)$

Differentiating w.r.t. x both the sides, we get $\frac{dy}{dx} = \cos(x + y) \times \left[1 + \frac{dy}{dx}\right]$

$\Rightarrow \frac{dy}{dx} = \cos(x + y) + \cos(x + y) \times \frac{dy}{dx}$

As it is given that $\frac{dy}{dx} = 0$ (at a point), so $0 = \cos(x + y) + \cos(x + y) \times 0$

$\Rightarrow \cos(x + y) = 0$

$\Rightarrow \cos^2(x + y) = 0$

$\Rightarrow 1 - \sin^2(x + y) = 0$ [$\because \sin^2\theta + \cos^2\theta = 1$

$\Rightarrow 1 - y^2 = 0$ [$\because y = \sin(x + y)$

$\Rightarrow y^2 = 1$

$\therefore y = \pm 1$.

23. (A) Let A(2, 6, 5), and B(3, p, 7); also let C(–4, 3, 4), and D(–1, 0, q).

Direction ratios of line AB : $3-2, p-6, 7-5$ i.e., $1, p-6, 2$.

Direction ratios of line CD : $-1-(-4), 0-3, q-4$ i.e., $3, -3, q-4$.

Since lines AB and CD are parallel, their d.r.'s will be proportional.

That is, $\frac{1}{3} = \frac{p-6}{-3} = \frac{2}{q-4}$

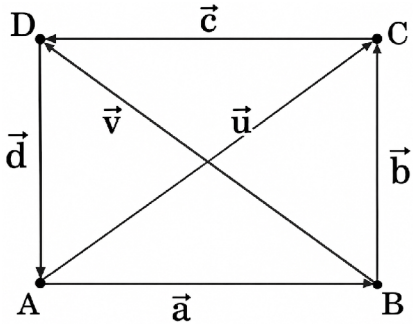
Taking $\frac{1}{3} = \frac{p-6}{-3}$ $\Rightarrow -1 = p - 6$ $\therefore p = 5$

Taking $\frac{1}{3} = \frac{2}{q-4}$ $\Rightarrow q - 4 = 6$ $\therefore q = 10$.

Now $\left(\frac{q}{p}\right)=\frac{10}{5}=2$.

OR

(B) Let the vectors representing the four sides of rectangle ABCD be $\overrightarrow{AB}=\vec{a}$, $\overrightarrow{BC}=\vec{b}$, $\overrightarrow{CD}=\vec{c}$ and $\overrightarrow{DA}=\vec{d}$.



Then $\overrightarrow{AC}=\vec{u}=\vec{a}+\vec{b}$ …(i) and $\overrightarrow{BD}=\vec{v}=\vec{b}+\vec{c}=\vec{b}-\vec{a}$ …(ii)

Solving (i) and (ii), we get $\vec{a}=\frac{\vec{u}-\vec{v}}{2}$ and $\vec{b}=\frac{\vec{u}+\vec{v}}{2}$.

Also $\vec{c}=-\vec{a}=\frac{\vec{v}-\vec{u}}{2}$ and $\vec{d}=-\vec{b}=\frac{-(\vec{u}+\vec{v})}{2}$.

24. Given that $|\vec{a}|=\sqrt{6}$, $\vec{a}.\vec{b}=1$ and $\vec{b}\times\vec{a}=3\hat{i}+4\hat{j}-2\hat{k}$.

Since $|\vec{b}\times\vec{a}|=|\vec{a}\times\vec{b}|=\sqrt{9+16+4}=\sqrt{29}$.

Also we know that, $|\vec{a}\times\vec{b}|^2+|\vec{a}.\vec{b}|^2=|\vec{a}|^2|\vec{b}|^2$

$\Rightarrow 29+1=6\times|\vec{b}|^2$

$\Rightarrow 30=6\times|\vec{b}|^2$

$\Rightarrow |\vec{b}|^2=\frac{30}{6}$

$\Rightarrow |\vec{b}|^2=5$

$\therefore |\vec{b}|=\sqrt{5}$.

25. For line AC $(3x+2y=10)$:

The shaded region lies to the right [away from the origin O(0, 0)] of the line. That is, origin is not included in feasible region. So, $3x+2y\geq 10$.

For line BC $(x+2y=6)$:

The shaded region lies below (towards the origin side) the line. That is, the origin is included in the feasible region. So, $x+2y\leq 6$.

Hence, the constraints of LPP are : $3x+2y\geq 10,\ x+2y\leq 6,\ x\geq 0,\ y\geq 0$.

Note that, the condition $x\geq 0$ is naturally satisfied since the minimum value of x in the region is at the intersection point C(2, 2). Though it is standard practice to include the non-negativity restrictions $x\geq 0,\ y\geq 0$.

25. **<u>FOR VISUALLY IMPAIRED</u>**

Since Z_{max} occurs at (p, q) and $\left(2p, \frac{q}{2}\right)$ both.

So, $3p+4q=3(2p)+4\left(\frac{q}{2}\right)$

$\Rightarrow 3p+4q=6p+2q$

$\Rightarrow 2q=3p$

$\Rightarrow 3p=2q$ or, $3p-2q=0$.

SECTION C

26. Let $A = PQ = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 2 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 2 & 2 \end{bmatrix}$.

Let $B = QP = \begin{bmatrix} 1 & 0 \\ 0 & 2 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 4 \\ 2 & 0 & 0 \end{bmatrix}$.

As $|A| = 4 \neq 0 \ \therefore A^{-1}$ exists.

Now $A^{-1} = \dfrac{\text{adj. } A}{|A|} = \dfrac{1}{4}\begin{bmatrix} 2 & 0 \\ -2 & 2 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 \\ -\frac{1}{2} & \frac{1}{2} \end{bmatrix}$.

Also note that, $|B| = 0 \ \therefore B^{-1}$ does not exist.

27. Let A be the area of the circular region covered by the drone and r be its radius.
Let h be the height of the drone at any time t.

Given that $\dfrac{dA}{dh} = 44 \text{ m}^2/\text{m}$ and $\dfrac{dr}{dt} = 6 \text{ m/s}$.

Since area of circle is, $A = \pi r^2$

$$\Rightarrow \frac{dA}{dt} = 2\pi r \times \frac{dr}{dt}$$

$$\Rightarrow \frac{dA}{dh} \times \frac{dh}{dt} = 2\pi r \times 6$$

$$\Rightarrow 44 \times \frac{dh}{dt} = 2\pi r \times 6$$

$$\Rightarrow \frac{dh}{dt} = \frac{12\pi r}{44}$$

$$\Rightarrow \left(\frac{dh}{dt}\right)_{\text{at r=14 m}} = \frac{3\pi \times 14}{11} = \frac{42\pi}{11} = \frac{42}{11} \times \frac{22}{7} = 12 \text{ m/s}.$$

28. (A) Given $x = \sin^3 t, \ y = \cos^3 t$

Differentiating w.r.t. t, $\dfrac{dx}{dt} = 3\sin^2 t \cos t, \ \dfrac{dy}{dt} = 3\cos^2 t(-\sin t)$

$$\therefore \frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx} = \frac{-3\cos^2 t \sin t}{3\sin^2 t \cos t} = -\cot t$$

Differentiating w.r.t. x, we get $\dfrac{d^2y}{dx^2} = \text{cosec}^2 t \times \dfrac{dt}{dx}$

$$\Rightarrow \frac{d^2y}{dx^2} = \text{cosec}^2 t \times \frac{1}{3\sin^2 t \cos t} = \frac{1}{3}(\text{cosec}^4 t \sec t) \quad \ldots(i)$$

At the point $\left(\dfrac{1}{2\sqrt{2}}, \dfrac{1}{2\sqrt{2}}\right)$, we have $\dfrac{1}{2\sqrt{2}} = \sin^3 t, \ \dfrac{1}{2\sqrt{2}} = \cos^3 t$

$$\Rightarrow \left(\frac{1}{\sqrt{2}}\right)^3 = \sin^3 t, \left(\frac{1}{\sqrt{2}}\right)^3 = \cos^3 t$$

$\Rightarrow \sin t = \frac{1}{\sqrt{2}}, \cos t = \frac{1}{\sqrt{2}}$

Clearly, $t = \frac{\pi}{4}$.

By (i), $\left(\frac{d^2y}{dx^2}\right)_{\text{at } t=\frac{\pi}{4}} = \frac{1}{3}\left[(\sqrt{2})^4(\sqrt{2})\right] = \frac{1}{3}\left[4\sqrt{2}\right] = \frac{4\sqrt{2}}{3}$.

OR

(B) $y = x^{\log x} + (\log x)^x$

$\Rightarrow y = e^{\log(x^{\log x})} + e^{\log[(\log x)^x]}$ [$\because x = e^{\log x}$

$\Rightarrow y = e^{\log x \log x} + e^{x\log(\log x)}$

$\Rightarrow y = e^{(\log x)^2} + e^{x\log(\log x)}$

Differentiating w.r.t. x, we get

$\frac{dy}{dx} = e^{(\log x)^2} \times 2\log x \times \frac{1}{x} + e^{x\log(\log x)} \times \left[x \times \frac{1}{\log x} \times \frac{1}{x} + \log(\log x) \times 1\right]$

$\Rightarrow \frac{dy}{dx} = x^{\log x} \times \frac{2\log x}{x} + (\log x)^x \times \left[\frac{1}{\log x} + \log(\log x)\right]$.

29. (A) We have $x\,dy + (y - x^2e^x)\,dx = 0$

$\Rightarrow x\,dy = -(y - x^2e^x)\,dx$

Rewriting the given D.E., we get $\frac{dy}{dx} + \frac{1}{x}y = x\,e^x$

On comparing with $\frac{dy}{dx} + P(x)\,y = Q(x)$, we get $P(x) = \frac{1}{x}$, $Q(x) = x\,e^x$.

So, I.F. $= e^{\int \frac{1}{x}dx} = e^{\log x} = x$.

The solution of D.E. is given by $y(x) = \int x\,(x\,e^x)\,dx + C$

$\Rightarrow x\,y = \int x^2e^x\,dx + C$ ...(i)

Taking $\int x^2e^x\,dx = x^2\int e^x\,dx - \int\left[\frac{d}{dx}(x^2)\int e^x\,dx\right]dx$

$= x^2e^x - \int 2x\,e^x dx$

$= x^2e^x - \left[2x\int e^x dx - \int\left\{\frac{d}{dx}(2x)\int e^x dx\right\}dx\right]$

$= x^2e^x - \left[2x\,e^x - \int 2\,e^x dx\right]$

$= x^2e^x - \left[2x\,e^x - 2\,e^x\right]$

By (i), we get the general solution as $x\,y = (x^2 - 2x + 2)e^x + C$.

It is given that $y(1) = 0$ i.e., when $x = 1$, $y = 0$.

So, $1 \times 0 = (1^2 - 2 \times 1 + 2)e^1 + C \Rightarrow C = -e$.

Hence, the particular solution is $x\,y = (x^2 - 2x + 2)e^x - e$.

OR

(B) We have $(x^2 + y^2)\,dy = xy\,dx$

Rewriting the given D.E., we get $\frac{dy}{dx}=\frac{xy}{x^2+y^2}$

$\Rightarrow \frac{dy}{dx}=\frac{\frac{y}{x}}{1+\left(\frac{y}{x}\right)^2}=f\left(\frac{y}{x}\right)$, hence it is a homogeneous D.E.

To solve, put $y=vx \Rightarrow \frac{dy}{dx}=v+x\frac{dv}{dx}$.

So, the D.E. reduces to $v+x\frac{dv}{dx}=\frac{v}{1+v^2}$

$\Rightarrow x\frac{dv}{dx}=\frac{v}{1+v^2}-v$

$\Rightarrow x\frac{dv}{dx}=\frac{-v^3}{1+v^2}$

$\Rightarrow \int\frac{1+v^2}{v^3}dv=-\int\frac{1}{x}dx$

$\Rightarrow \int\left(\frac{1}{v^3}+\frac{1}{v}\right)dv=-\int\frac{1}{x}dx$

$\Rightarrow -\frac{1}{2v^2}+\log|v|=-\log|x|+C$

$\Rightarrow -\frac{x^2}{2y^2}+\log\left|\frac{y}{x}\right|=-\log|x|+C$

$\Rightarrow -\frac{x^2}{2y^2}+\log|y|-\log|x|=-\log|x|+C$

$\Rightarrow -\frac{x^2}{2y^2}+\log|y|=C$

It is given that $y(0)=1$ i.e., when $x=0,\ y=1$.

So, $-\frac{0^2}{2\times1^2}+\log|1|=C \Rightarrow C=0$.

Hence, the particular solution is $-\frac{x^2}{2y^2}+\log|y|=0$ or $\log|y|=\frac{x^2}{2y^2}$ or $y=e^{\frac{x^2}{2y^2}}$.

30. (A) Rewriting the given line (L) : $\frac{x-2}{1}=\frac{y+\frac{3}{2}}{2}=\frac{z-2}{-1}=\lambda$.

The direction ratios of the line (L) are 1, 2, –1.

A random point on the line (L) can be taken as $P\left(\lambda+2,\ 2\lambda-\frac{3}{2},\ 2-\lambda\right)$.

Let A(2, 1, 4). Then the direction ratios of the line AP will be $\lambda,\ 2\lambda-\frac{5}{2},\ -2-\lambda$.

When AP is perpendicular to the line (L), we shall have $1\times\lambda+2\left(2\lambda-\frac{5}{2}\right)+(-1)(-2-\lambda)=0$

$\Rightarrow \lambda+4\lambda-5+2+\lambda=0$

$\Rightarrow 6\lambda = 3$

$\Rightarrow \lambda = \frac{1}{2}$.

Hence, the required foot of the perpendicular is $P\left(\frac{5}{2}, -\frac{1}{2}, \frac{3}{2}\right)$.

Equation of line (AP) : $\frac{x-2}{\frac{1}{2}} = \frac{y-1}{-\frac{3}{2}} = \frac{z-4}{-\frac{5}{2}}$ i.e., $\frac{x-2}{1} = \frac{y-1}{-3} = \frac{z-4}{-5}$ i.e., $x-2 = \frac{1-y}{3} = \frac{4-z}{5}$.

OR

(B) Let (L_1) : $\frac{x-1}{2} = \frac{y-1}{2} = \frac{z-2}{1}$; its direction ratios are 2, 2, 1.

Let (L_2) : $\frac{x-5}{2} = y - \frac{1}{2} = \frac{3-z}{2}$ i.e., $\frac{x-5}{2} = \frac{y-\frac{1}{2}}{1} = \frac{z-3}{-2}$; its direction ratios are 2, 1, –2.

Let θ be the required angle between these lines.

$\therefore \cos\theta = \frac{2\times 2 + 2\times 1 + 1\times(-2)}{\sqrt{2^2+2^2+1^2}\sqrt{2^2+1^2+(-2)^2}} = \frac{4}{\sqrt{9}\sqrt{9}} = \frac{4}{9}$

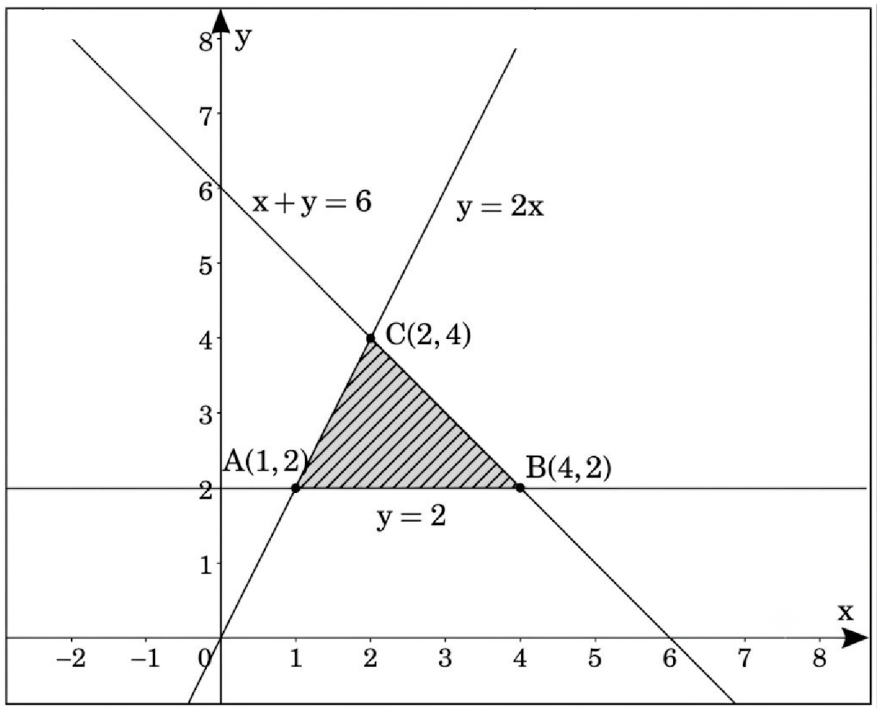
$\Rightarrow \theta = \cos^{-1}\left(\frac{4}{9}\right)$.

Further, it has been given that the required line (*say* L) is passing through the point (1, 1, 1) and perpendicular to both lines.

Clearly, line (L) will be in the direction of vector $\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 2 & 1 \\ 2 & 1 & -2 \end{vmatrix} = -5\hat{i} + 6\hat{j} - 2\hat{k}$.

Hence, the Cartesian equation of line (L) will be $\frac{x-1}{-5} = \frac{y-1}{6} = \frac{z-1}{-2}$ i.e., $\frac{x-1}{5} = \frac{y-1}{-6} = \frac{z-1}{2}$.

31. Refer the graph shown below.



Feasible region is bounded with three vertices A(1, 2), B(4, 2), C(2, 4).
We need to maximize $Z = 40x + 30y$.

Now $Z_A = 40\times1 + 30\times2 = 100$,

$Z_B = 40\times4 + 30\times2 = 220$,

$Z_C = 40\times2 + 30\times4 = 200$.

So, maximum value of Z is 220 at the point B(4, 2).

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The vertices of the feasible region are A(20, 0), B(40, 0), C(30, 10), D(0, 20) and E(0, 10).
We need to minimize and maximize $Z = 50x - 20y$.

Now $Z_A = 50\times20 - 20\times0 = 1000$,

$Z_B = 50\times40 - 20\times0 = 2000$,

$Z_C = 50\times30 - 20\times10 = 1300$,

$Z_D = 50\times0 - 20\times20 = -400$,

$Z_E = 50\times0 - 20\times10 = -200$.

Clearly, the minimum value of objective function Z is –400 at D(0, 20), and the maximum value of objective function Z is 2000 at B(40,0).

SECTION D

32. (A) Relation R is defined on $A = N\times N$ such that (a, b) R (c, d) if and only if $a^2 + d^2 = b^2 + c^2$.

Reflexive : Let $(p, q) \in A = N\times N$.

Then it is clear that $p, q \in N$.

So, $p^2 + q^2 = q^2 + p^2$

$\Rightarrow (p, q) R (p, q)$.

Therefore, R is reflexive relation.

Symmetric : Let $(p, q), (r, s) \in A = N\times N$ such that $(p, q) R (r, s)$.

So, $p^2 + s^2 = q^2 + r^2$

$\Rightarrow q^2 + r^2 = p^2 + s^2$

$\Rightarrow r^2 + q^2 = s^2 + p^2$

$\Rightarrow (r, s) R (p, q)$.

Therefore, R is symmetric relation.

Transitive : Let $(p, q), (r, s), (u, v) \in A = N\times N$ such that $(p, q) R (r, s)$ and $(r, s) R (u, v)$.

So, $p^2 + s^2 = q^2 + r^2$ ...(i) and $r^2 + v^2 = s^2 + u^2$ ...(ii)

Adding (i) and (ii), we get $p^2 + s^2 + r^2 + v^2 = q^2 + r^2 + s^2 + u^2$

$\Rightarrow p^2 + (s^2 + r^2) + v^2 = q^2 + (r^2 + s^2) + u^2$

$\Rightarrow p^2 + v^2 = q^2 + u^2$

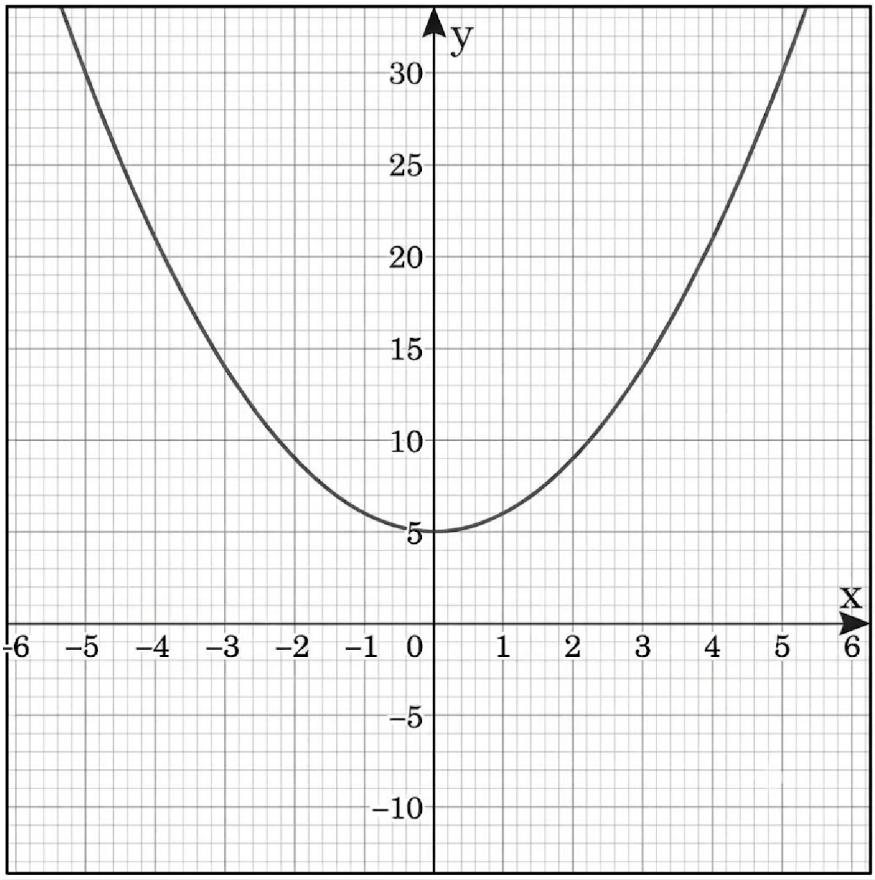
$\Rightarrow (p, q) R (u, v)$.

Therefore, R is transitive relation.
So, R is reflexive, symmetric and transitive.
Hence R is an equivalence relation.

OR

(B) Given $R : (-\infty, \infty) \to [0, \infty)$ is defined as $R = \{(x, y) : y = x^2 + 5\}$.

Take $y = x^2 + 5$ for $x \in (-\infty, \infty)$.
Refer the graph shown below.



Yes, R is a function as the graph shows every element of the set $(-\infty, \infty)$ has a unique image in the set $[0, \infty)$.

Let us denote this function by $f : (-\infty, \infty) \to [0, \infty)$ as $f(x) = x^2 + 5$.

• Injective function: Since $f(1) = 1^2 + 5 = 6$, $f(-1) = (-1)^2 + 5 = 6$ i.e., $f(1) = f(-1)$ but $1 \neq -1$.
Thus f is a many-one function, so not injective.

• Surjective function: Range of $f(x) = [5, \infty) \neq$ codomain. So, it is not surjective.
Note that, codomain is $[0, \infty)$.

Alternatively, make a correct graph as above.
Clearly, R is a function as graph depicts every element of the set $(-\infty, \infty)$ has a unique image in the set $[0, \infty)$.
Let us denote this function by $f : (-\infty, \infty) \to [0, \infty)$ as $f(x) = x^2 + 5$.

• Injective function: Let $a, b \in (-\infty, \infty)$, such that $f(a) = f(b)$.

Then $a^2 + 5 = b^2 + 5$
$\Rightarrow a^2 = b^2$
$\Rightarrow a = \pm b$
So, f is not injective.

• Surjective function: For $f(x) = 1$, we get $1 = x^2 + 5 \Rightarrow x^2 = -4$, which is not possible for all values of $x \in (-\infty, \infty)$.
That is, there is no pre-image of $1 \in$ codomain $= [0, \infty)$ in the domain $(-\infty, \infty)$.
So, f is not surjective.

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For every element $a \in (-\infty, \infty)$, we shall have $a^2 \in [0, \infty)$, therefore $a^2 + 7 \in [7, \infty) \subseteq [0, \infty)$.
Hence every element of the set $(-\infty, \infty)$ has a unique image in the set $[0, \infty)$.
So, $R : (-\infty, \infty) \to [0, \infty)$ given by $R = \{(x, y) : y = x^2 + 7\}$ is a function.
Let us denote this function by $f : (-\infty, \infty) \to [0, \infty)$ as $f(x) = x^2 + 7$.

- Injective function: Since $f(1) = 1^2 + 7 = 8$, $f(-1) = (-1)^2 + 7 = 8$ i.e., $f(1) = f(-1)$ but $1 \neq -1$.

Thus f is not injective.

- Surjective function: Range of $f(x) = [7, \infty) \neq$ codomain. So, it is not surjective.

Note that, codomain is $[0, \infty)$.
Thus, the function f is neither injective nor surjective, and hence not bijective.

33. Let $I = \int_{-a}^{a} \sqrt{\frac{a-x}{a+x}}\, dx$

$$\Rightarrow I = \int_{-a}^{a} \sqrt{\frac{a-x}{a+x} \times \frac{a-x}{a-x}}\, dx$$

$$\Rightarrow I = \int_{-a}^{a} \frac{a-x}{\sqrt{a^2 - x^2}}\, dx$$

$$\Rightarrow I = \int_{-a}^{a} \frac{a}{\sqrt{a^2 - x^2}}\, dx - \int_{-a}^{a} \frac{x}{\sqrt{a^2 - x^2}}\, dx$$

Take $f(x) = \frac{a}{\sqrt{a^2 - x^2}}$ and $g(x) = \frac{x}{\sqrt{a^2 - x^2}}$

$\because f(-x) = \frac{a}{\sqrt{a^2 - (-x)^2}} = \frac{a}{\sqrt{a^2 - x^2}} = f(x)$ and $g(-x) = \frac{-x}{\sqrt{a^2 - (-x)^2}} = \frac{-x}{\sqrt{a^2 - x^2}} = -g(x)$.

Using $\int_{-a}^{a} f(x)\, dx = \begin{cases} 2\int_0^a f(x)\, dx, & \text{if } f(-x) = f(x) \\ 0, & \text{if } f(-x) = -f(x) \end{cases}$, we get $I = 2\int_0^a \frac{a}{\sqrt{a^2 - x^2}}\, dx - 0$

$$\Rightarrow I = 2a\left[\sin^{-1}\frac{x}{a}\right]_0^a$$

$$\Rightarrow I = 2a\left[\sin^{-1} 1 - \sin^{-1} 0\right] = 2a\left[\frac{\pi}{2} - 0\right]$$

$\therefore I = a\pi$.

Alternatively, let $I = \int_{-a}^{a} \sqrt{\frac{a-x}{a+x}}\, dx$

Put $x = a\cos 2\theta \Rightarrow dx = -2a\sin 2\theta\, d\theta$.

When $x = -a \Rightarrow \cos 2\theta = -1 \Rightarrow 2\theta = \pi \Rightarrow \theta = \frac{\pi}{2}$.

When $x = a \Rightarrow \cos 2\theta = 1 \Rightarrow 2\theta = 0 \Rightarrow \theta = 0$.

$$\therefore I = -2a\int_{\frac{\pi}{2}}^{0} \sqrt{\frac{a - a\cos 2\theta}{a + a\cos 2\theta}} \sin 2\theta\, d\theta$$

$$\Rightarrow I = 2a\int_0^{\frac{\pi}{2}} \sqrt{\frac{1-\cos 2\theta}{1+\cos 2\theta}}\sin 2\theta\, d\theta = 2a\int_0^{\frac{\pi}{2}} \sqrt{\frac{2\sin^2\theta}{2\cos^2\theta}}\sin 2\theta\, d\theta = 2a\int_0^{\frac{\pi}{2}} \tan\theta \sin 2\theta\, d\theta$$

$$\Rightarrow I = 2a\int_0^{\frac{\pi}{2}} \frac{\sin\theta}{\cos\theta}\times 2\sin\theta\cos\theta\, d\theta$$

$$\Rightarrow I = 2a\int_0^{\frac{\pi}{2}} 2\sin^2\theta\, d\theta$$

$$\Rightarrow I = 2a\int_0^{\frac{\pi}{2}} (1-\cos 2\theta)\, d\theta$$

$$\Rightarrow I = 2a\left[\theta - \frac{\sin 2\theta}{2}\right]_0^{\frac{\pi}{2}}$$

$$\Rightarrow I = 2a\left[\left(\frac{\pi}{2} - \frac{\sin\pi}{2}\right) - \left(0 - \frac{\sin 0}{2}\right)\right]$$

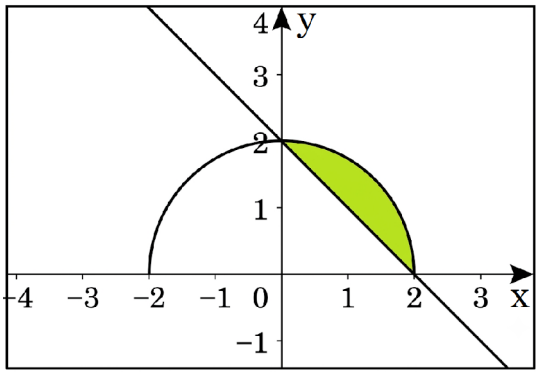
$$\Rightarrow I = 2a\left[\left(\frac{\pi}{2} - 0\right) - 0\right]$$

$\therefore I = a\pi$.

34. (A) Consider $y = \sqrt{4-x^2}$, $y = 0$, $x = 2 - y$.

$y = \sqrt{4-x^2}$ is a semicircle with $r = 2$ and its centre is at (0, 0).

Also $x = 2 - y \Rightarrow \frac{x}{2} + \frac{y}{2} = 1$ is a straight line whose intercepts on x and y both axes are 2.



$$\therefore \text{ Required area} = \int_0^2 \left[\sqrt{4-x^2} - (2-x)\right] dx$$

$$= \left[\frac{x}{2}\sqrt{4-x^2} + \frac{4}{2}\sin^{-1}\frac{x}{2} - \frac{(2-x)^2}{2(-1)}\right]_0^2$$

$$= \left[\frac{2}{2}\sqrt{4-4} + 2\sin^{-1}1 - \frac{(0)^2}{2(-1)}\right] - \left[0\sqrt{4-0} + 2\sin^{-1}0 - \frac{(2)^2}{2(-1)}\right]$$

$$= \left[2\times\frac{\pi}{2}\right] - [0+0+2]$$

$= (\pi - 2)$ Sq. units.

OR

(B) $y^2 = 2x$ is a parabola with vertex at (0, 0).

For the point of intersection of parabola $y^2 = 2x$ and the line $x - 2y = 0$:

Put $x = 2y$ in $y^2 = 2x$.

$\therefore y^2 = 2(2y)$

$\Rightarrow y^2 - 4y = 0$

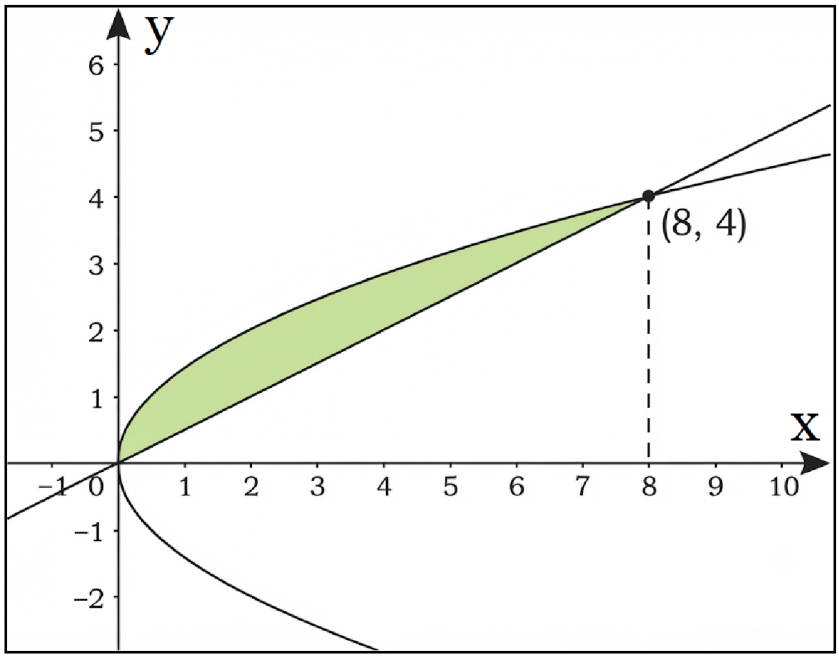
$\Rightarrow y(y-4) = 0$

Either $y = 0$ or $(y - 4) = 0$

That is, $y = 0$ or $y = 4$.

Hence, $x = 0$ or $x = 8$

$\therefore$ Points of intersection are $(0, 0), (8, 4)$.



$\therefore$ Required area $= \int_0^8 \left[\sqrt{2x} - \frac{x}{2}\right] dx$

$$= \left[\frac{2}{3}\sqrt{2} \times x\sqrt{x} - \frac{1}{4}x^2\right]_0^8$$

$$= \left[\frac{2}{3}\sqrt{2} \times 8\sqrt{8} - \frac{1}{4} \times 8^2\right] - \left[\frac{2}{3}\sqrt{2} \times 0\sqrt{0} - \frac{1}{4} \times 0^2\right]$$

$$= \left[\frac{16}{3}\sqrt{2} \times 2\sqrt{2} - \frac{1}{4} \times 64\right] - 0$$

$$= \left[\frac{16 \times 4}{3} - 16\right]$$

$$= 16\left[\frac{4}{3} - 1\right]$$

$$= \frac{16}{3} \text{ Sq. units.}$$

FOR VISUALLY IMPAIRED

34. (A) $25x^2 + 16y^2 = 400$ is an ellipse with standard form of equation given as $\frac{x^2}{16} + \frac{y^2}{25} = 1$.

Then $y = \frac{5}{4}\sqrt{16 - x^2}$.

Also, note that the ellipse is symmetrical about both the axes.

$\therefore$ Required area $= 4\int_0^4 \frac{5}{4}\sqrt{16 - x^2}\, dx$

$$= 5\left[\frac{x}{2}\sqrt{16-x^2} + \frac{16}{2}\sin^{-1}\frac{x}{4}\right]_0^4$$

$$= 5\left\{\left[\frac{4}{2}\sqrt{16-16} + 8\sin^{-1}1\right] - \left[0\sqrt{16-0} + 8\sin^{-1}0\right]\right\}$$

$$= 5\left\{\left[2\times 0 + 8\times\frac{\pi}{2}\right] - 0\right\}$$

$= 20\pi$ Sq. units.

OR

(B) $y^2 = 8x$ is a parabola, and the area of the region above the x-axis, below the parabola, between the lines $x = 2$ and $x = 5$ is given by :

$$\text{Required area} = \int_2^5 \sqrt{8x}\, dx$$

$$= \sqrt{8}\left[\frac{2x\sqrt{x}}{3}\right]_2^5$$

$$= \frac{4\sqrt{2}}{3}\left[5\sqrt{5} - 2\sqrt{2}\right]$$

$$= \frac{4}{3}\left[5\sqrt{10} - 4\right] \text{ Sq. units.}$$

35. Let the events be

E_1 : user uses Alpha smartwatch;

E_2 : user uses Beta smartwatch;

E_3 : user uses Gamma smartwatch;

E_4 : user uses Delta smartwatch;

A : user gets a false heart-rate alert.

Clearly, $\{E_1, E_2, E_3, E_4\}$ is a partition of the sample space.

Then $P(E_1) = 0.45$, $P(E_2) = 0.3$, $P(E_3) = 0.15$, $P(E_4) = 0.1$.

Now $P(A \mid E_1) = 0.01$, $P(A \mid E_2) = 0.02$, $P(A \mid E_3) = 0.05$, $P(A \mid E_4) = 0.01$.

(i) By the theorem of Total Probability, the total probability that a randomly selected user gets a false alert $= P(A) = P(E_1)P(A \mid E_1) + P(E_2)P(A \mid E_2) + P(E_3)P(A \mid E_3) + P(E_4)P(A \mid E_4)$

$$= (0.45)(0.01) + (0.3)(0.02) + (0.15)(0.05) + (0.1)(0.01)$$

$$= (0.0045) + (0.006) + (0.0075) + (0.001)$$

$$= 0.019.$$

(ii) By Bayes' theorem, the probability that the false alert came from a Beta smartwatch

$$P(E_2 \mid A) = \frac{P(E_2)P(A \mid E_2)}{P(E_1)P(A \mid E_1) + P(E_2)P(A \mid E_2) + P(E_3)P(A \mid E_3) + P(E_4)P(A \mid E_4)}$$

$$\Rightarrow P(E_2 \mid A) = \frac{0.3\times 0.02}{0.019} = \frac{0.006}{0.019}$$

$$\therefore P(E_2 \mid A) = \frac{6}{19}.$$

SECTION E

36. We have been given

$l_1 : \vec{r} = (\lambda+1)\hat{i} + (3\lambda+2)\hat{j} + (3-\lambda)\hat{k} \quad \Rightarrow \vec{r} = \hat{i} + 2\hat{j} + 3\hat{k} + \lambda(\hat{i} + 3\hat{j} - \hat{k})$;

$l_2 : \vec{r} = 2(\mu+1)\hat{i} + (3\mu-1)\hat{j} + (\mu+1)\hat{k} \quad \Rightarrow \vec{r} = 2\hat{i} - \hat{j} + \hat{k} + \mu(2\hat{i} + 3\hat{j} + \hat{k})$;

$l_3 : \vec{r} = (\gamma+3)\hat{i} + (3\gamma-1)\hat{j} + (1-\gamma)\hat{k} \quad \Rightarrow \vec{r} = 3\hat{i} - \hat{j} + \hat{k} + \gamma(\hat{i} + 3\hat{j} - \hat{k})$

The direction ratios of l_1, l_2 and l_3 are respectively 1, 3, –1; 2, 3, 1; and 1, 3, –1.

(i) Since the direction ratios of l_1 and l_2 are not proportional i.e., $\frac{1}{2} \neq \frac{3}{3} \neq \frac{-1}{1}$.

Hence, the lines l_1 and l_2 are not parallel to each other.

(ii) Since the direction ratios of l_1 and l_3 are proportional i.e., $\frac{1}{1} = \frac{3}{3} = \frac{-1}{-1}$.

Hence, the lines l_1 and l_3 are parallel to each other.

(iii) (A) For l_1, $\vec{a}_1 = \hat{i} + 2\hat{j} + 3\hat{k}$, $\vec{b}_1 = \hat{i} + 3\hat{j} - \hat{k}$;

For l_2, $\vec{a}_2 = 2\hat{i} - \hat{j} + \hat{k}$, $\vec{b}_2 = 2\hat{i} + 3\hat{j} + \hat{k}$.

Now $\vec{a}_2 - \vec{a}_1 = \hat{i} - 3\hat{j} - 2\hat{k}$; $\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 3 & -1 \\ 2 & 3 & 1 \end{vmatrix} = 6\hat{i} - 3\hat{j} - 3\hat{k}$.

Required S.D. between the lines l_1 and $l_2 = \dfrac{|(\vec{a}_2 - \vec{a}_1).(\vec{b}_1 \times \vec{b}_2)|}{|(\vec{b}_1 \times \vec{b}_2)|}$

$$= \frac{|(\hat{i} - 3\hat{j} - 2\hat{k}).(6\hat{i} - 3\hat{j} - 3\hat{k})|}{|6\hat{i} - 3\hat{j} - 3\hat{k}|}$$

$$= \frac{|6+9+6|}{\sqrt{36+9+9}} = \frac{21}{\sqrt{54}} = \frac{21}{3\sqrt{6}} = \frac{7}{\sqrt{6}} = \frac{7\sqrt{6}}{6} \text{ units}.$$

OR

(iii) (B) For l_1, $\vec{a}_1 = \hat{i} + 2\hat{j} + 3\hat{k}$, $\vec{b}_1 = \hat{i} + 3\hat{j} - \hat{k}$;

For l_3, $\vec{a}_3 = 3\hat{i} - \hat{j} + \hat{k}$, $\vec{b}_3 = \hat{i} + 3\hat{j} - \hat{k}$.

Since $\vec{b}_1 = \vec{b}_3$, so lines $l_1 \parallel l_3$. Take $\vec{b}_1 = \vec{b} = \hat{i} + 3\hat{j} - \hat{k}$

You may also refer to sub-part **(ii)** for proof of lines l_1 and l_3 being parallel.

Now $\vec{a}_3 - \vec{a}_1 = 2\hat{i} - 3\hat{j} - 2\hat{k}$; $(\vec{a}_3 - \vec{a}_1) \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -3 & -2 \\ 1 & 3 & -1 \end{vmatrix} = 9\hat{i} + 0\hat{j} + 9\hat{k}$.

Required S.D. between the lines l_1 and $l_3 = \dfrac{|(\vec{a}_3 - \vec{a}_1) \times \vec{b}|}{|\vec{b}|}$

$$= \frac{\sqrt{81+0+81}}{\sqrt{1+9+1}} = \frac{9\sqrt{2}}{\sqrt{11}} = 9\sqrt{\frac{2}{11}} = \frac{9\sqrt{22}}{11} \text{ units}.$$

37. **(i)** $W(x) = 35 + 1.6x - 0.02x^2$

$\Rightarrow W'(x) = 1.6 - 0.04x$

For stationary point, $W'(x) = 0$, $1.6 - 0.04x = 0$

$\Rightarrow 0.04x = 1.6$

$\Rightarrow x = 40$.

Therefore, $x = 40$ crores at the stationary point of W(x).

(ii) Since $W'(x) = 1.6 - 0.04x$

$\Rightarrow W''(x) = -0.04 < 0$ (at $x = 40$).

So, W(x) is maximum at $x = 40$.

Thus, the WQI is maximum when ₹40 crores are spent on the revival.

Also the maximum value of the WQI $= W(40) = 35 + 1.6 \times 40 - 0.02(40)^2 = 35 + 64 - 32 = 67$.

(iii) (A) If the investment is ₹35 crores, then the $WQI = W(35) = 35 + 1.6 \times 35 - 0.02(35)^2$

$= 35 + 56 - 24.5 = 66.5$

$\because$ Maximum value of WQI is 67. Refer to sub-part **(ii)**.

Then the difference between WQI_{max} and $W(35)$ is $67 - 66.5 = 0.5$.

So, the WQI is short by 0.5 units of its maximum level.

OR

(iii) (B) $W(60) = 35 + 1.6 \times 60 - 0.02(60)^2 = 35 + 96 - 72 = 59$.

$\because$ Maximum value of WQI is 67. Refer to sub-part **(ii)**.

Clearly, $W(60) = 59$ is less than the maximum value 67 of WQI, even though the investment increased from 40 crores to 60 crores. This shows that the returns are diminishing.

38. Taking the price of each kg of apples, oranges, and pomegranate as ₹x, ₹y, and ₹z respectively.

Clearly, we have $5x + 10y + 5z = 2750$, $10x + 10y + 5z = 3500$, $8x + 15y + 7z = 4100$.

(i) Then the matrix form is $AX = B$, where $A = \begin{pmatrix} 5 & 10 & 5 \\ 10 & 10 & 5 \\ 8 & 15 & 7 \end{pmatrix}$, $X = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$, $B = \begin{pmatrix} 2750 \\ 3500 \\ 4100 \end{pmatrix}$.

So, adjoint of the coefficient matrix is $\text{adj.}A = \begin{pmatrix} -5 & 5 & 0 \\ -30 & -5 & 25 \\ 70 & 5 & -50 \end{pmatrix}$.

(ii) Since $|A| = 25 \neq 0 \quad \therefore A^{-1}$ exists.

$\because A^{-1} = \dfrac{\text{adj.}A}{|A|} \qquad \Rightarrow A^{-1} = \dfrac{1}{25}\begin{pmatrix} -5 & 5 & 0 \\ -30 & -5 & 25 \\ 70 & 5 & -50 \end{pmatrix}$

As $AX = B$ implies, $X = A^{-1}B$

$\Rightarrow X = \dfrac{1}{25}\begin{pmatrix} -5 & 5 & 0 \\ -30 & -5 & 25 \\ 70 & 5 & -50 \end{pmatrix}\begin{pmatrix} 2750 \\ 3500 \\ 4100 \end{pmatrix}$

$\Rightarrow \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 150 \\ 100 \\ 200 \end{pmatrix}$

By equality of matrices, we get $x = 150$, $y = 100$, $z = 200$.

Hence, the price of apples, oranges, and pomegranate are respectively ₹150 per kg, ₹100 per kg, ₹200 per kg.

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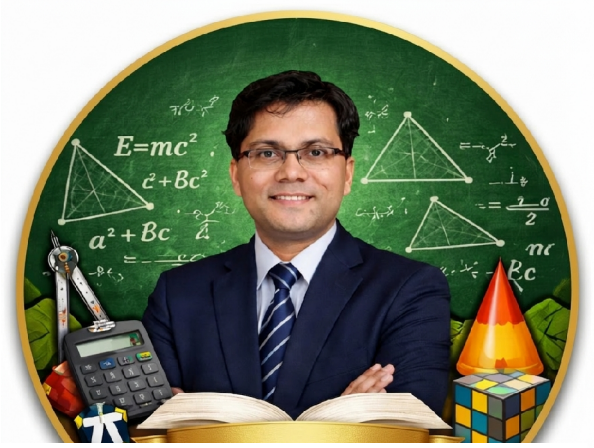
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ABOUT THE AUTHOR

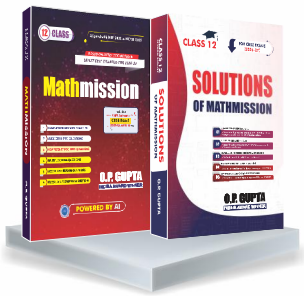
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His resources have helped students and teachers for a long time across the country. He has contributed in CBSE Question Bank (issued in April 2021). Mr Gupta has been invited by many educational institutions for hosting sessions for the students of senior classes. Being qualified as an electronics & communications engineer, he has pursued his graduation later on with mathematics from University of Delhi due to his passion towards mathematics. He has been honored with the prestigious INDIRA AWARD by the Govt. of Delhi for excellence in education.

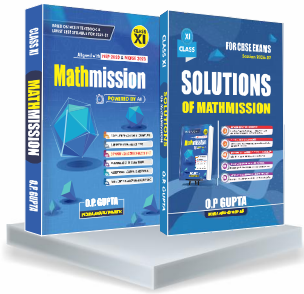
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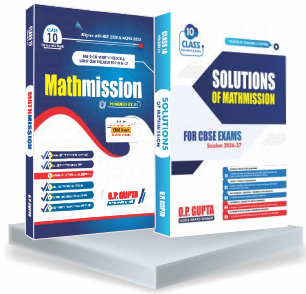
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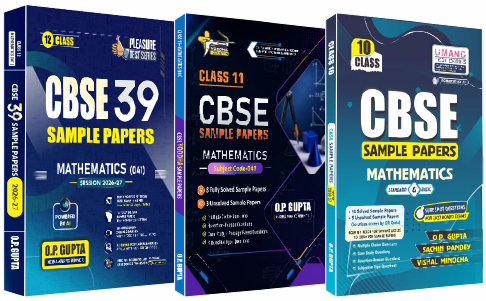


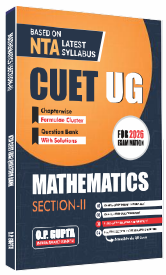
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